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CALCULUS.

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295. Proposed by C. E. FLANAGAN, Wheeling, W. Va.

A hawk can fly v feet per second, a hare can run v' feet per second. The hawk, when a feet vertically above the hare, gives chase and catches the hare when the hare has run b feet. Find the length of the curve of pursuit. [Echols' *Differential and Integral Calculus*, page 253, Ex. 20.]

I. Solution by FRANK LOXLEY GRIFFIN, Ph. D., Williams College.

The time that the hare runs is clearly b/v' seconds, which is also the time of the hawk's flight; thus, S, the length of the curve of pursuit, is vb/v' feet.

Remark. For the given area a to be consistent with v, v' and b there is a certain necessary condition (found below), which if fulfilled renders part of the data superfluous, as shown by the preceding solution. Further, if b be supposed unknown, we can still find S in terms of a, v, v'.

Let the hare's path be taken as the X-axis starting from (0, 0) so that the hawk starts from $(0, a)$; and let their coordinates be, respectively, $(X, 0)$ and (x, y) after t seconds. Then the hawk has flown a distance s:

$$X=x-y\frac{dx}{dy}=v's/v\ldots\text{(I)}.$$

Differentiating (I) with respect to y gives: $\dfrac{v'}{v}\dfrac{ds}{dy}=-y\dfrac{d^2x}{dy^2}$; in which put $\dfrac{dx}{dy}=p$ and $\dfrac{v'}{v}=k<1$, noting that $\dfrac{ds}{dy}=-\sqrt{(1+p^2)}$, and separate variables: $kdy/y=dp/\sqrt{(1+p^2)}$. Integrating:

$$\log[p+\sqrt{(1+p^2)}]=\log(cy)^k,\ \text{ or }\ p+\sqrt{(1+p^2)}=(cy)^k\ldots\text{(II)}.$$

In (II) invert both members and rationalize denominators: $\sqrt{(1+p^2)}-p=(cy)^{-k}$, whence $2p=(cy)^k-(cy)^{-k}$.

Again separating and integrating we have

$$2c(x+d)=\frac{(cy)^{1+k}}{1+k}-\frac{(cy)^{1-k}}{1-k}\ldots\text{(III)}.$$

The constants c, d are determined by the fact that, when $x=0$, $y=a$ and $p=0$. Thus (II) gives $c=1/a$, and (III) gives $d=-ak/(1-k^2)$. We find b from the fact that $(b, 0)$ must satisfy (III): $b=-d=ak/(1-k^2.)$ [This is the above-mentioned necessary condition.]

Since $S=vb/v'=b/k$, we have $S=a/(1-k^2)$ or $v'^2a/v^2-v'^2)$.

Also solved by G. B. M. Zerr, E. L. Sherwood, and V. M. Spunar.

II. Solution by C. E. FLANAGAN.

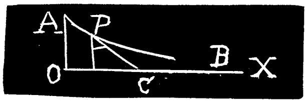
This problem as above stated is not a Calculus problem at all, but one of simple proportion; for, plainly, if s=the length of the hawk's path, we have, $s:b=v:v'$. Hence $s=b\frac{v}{v'}$. When b is *unknown* and only a, v, and v' are given, then we have a Calculus problem to solve, and under these conditions I offer the following

Solution. Let s=length of hawk's path and $\frac{v'}{v}=n$. From similar triangles we have,

$$-dy:y=dx:ns-x.$$

$\therefore ns-x=-y\frac{dx}{dy}$; $\therefore nds-dx=-yd\left(\frac{dx}{dy}\right)-\frac{dx}{dy}$; or since $\frac{dx}{dy}dy=dx$, $nds=-yd\left(\frac{dx}{dy}\right)$. But $ds=\sqrt{1+\frac{dx^2}{dy^2}}dy$. $\therefore n\sqrt{1+\frac{dx^2}{dy^2}}dy=-yd\left(\frac{dx}{dy}\right)$;

$$\therefore -n\frac{dy}{y}=\frac{d\left(\frac{dx}{dy}\right)}{\sqrt{1+\frac{dx^2}{dy^2}}}.$$



Integrating, $c-n\log y=\log\left(\frac{dx}{dy}+\sqrt{1+\frac{dx^2}{dy^2}}\right)$. Now when $y=a$, $\frac{dx}{dy}=0$; $\therefore c=n\log a$.

$$\therefore n\log a-n\log y=\log\left(\frac{dx}{dy}+\sqrt{1+\frac{dx^2}{dy^2}}\right);$$

$$\log\left(\frac{a}{y}\right)^n=\log\left(\frac{dx}{dy}+\sqrt{1+\frac{dx^2}{dy^2}}\right);$$

$$\frac{a^n}{y^n}=\frac{dx}{dy}+\sqrt{1+\frac{dx^2}{dy^2}};\quad \therefore\sqrt{1+\frac{dx^2}{dy^2}}=\frac{a^n}{y^n}-\frac{dx}{dy};$$

$$1+\frac{dx^2}{dy^2}=\frac{a^{2n}}{y^{2n}}-2\frac{a^n}{y_n}\frac{dx}{dy}+\frac{dx^2}{dy^2};\quad 2\frac{dx}{dy}=\frac{a^n}{y^n}-\frac{y^n}{a^n};$$

$2dx=a^ny^{-n}dy-a^{-n}y^ndy$. Hence, integrating, we get

$$2x+c'=\frac{a^n}{1-n}y^{1-n}-\frac{a^{-n}}{1+n}y^{1+n}.$$

Now when $x=0$, $y=a$.

$$\therefore c'=\frac{a}{1-n}-\frac{a}{1+n}=\frac{2an}{1-n^2};$$

$\therefore 2x+\dfrac{2an}{1-n^2}=\dfrac{a^n}{1-n}y^{1-n}-\dfrac{a^{-n}}{1+n}y^{1+n}$, which is the equation of the curve of pursuit. From this equation, when $x=ns$, $y=0$, we get $2ns+\dfrac{2an}{1-n^2}=0$; $\therefore s=-\dfrac{a}{1-n^2}$ =length of the hawk's path; and $b=ns=-\dfrac{an}{1-n^2}$ =length of the hare's path. The minus sign before the fraction shows the distances are measured from B toward A and O.

To get the formula given by Mr. Echols, we have

$$n^2=\frac{b^2}{s^2}=1+\frac{a}{s}.\quad \therefore s^2-as=b^2;\quad \therefore s=\pm\sqrt{\frac{a^2}{4}+b^2}+\frac{a}{2}.$$

NOTE.—We have published these two solutions for the purpose of comparison. Mr. Flanagan contends that the problem does not belong to the Calculus, which in a sense is right. Yet had b been the unknown quantity, the problem would certainly have been one involving Calculus. The fact that b is given, imposes a restriction on the given quantities.

Of the problem, "A dog and duck are at the opposite ends of the diameter of a circular pond. How far must the dog swim to catch the duck, the dog swimming n times as fast as the duck and directly towards it at all times, $n>1$," Mr. Flanagan offers the following solution, which we publish for criticism:

Let the diameter AB be an inextensible string and P and P' two moving pencils. The pencils move in opposite directions around the circle. They are together at B at the start. They move, say, at equal speeds. The pencil P' is fastened to the end of the string, but the string moves freely against the end of the pencil P. The heavy pellet A is pulled to the position D along the dotted line whose length at any moment is equal to S, the line DP being tangent to the path of the pellet. It is plain that $PP'=S$ and that $DP=2r-s$. Now when the pellet has reached the circumference $DP=2r-s=0$. Therefore, $s=2r$ when the pursued travels half as fast as the pursuer. In like manner it can be shown that the length of the path of the pursuer is always equal to the diameter of the circle when the pursuer travels as fast or faster than the pursued.